Imperial College London
Civil Engineering 2
Mathematics

Autumn Term Progress Test
Thursday 29th November 2007 4.00-5.00
Answer all questions

1. Show that $y = e^x$ is a solution of the differential equation

$$x\frac{d^2y}{dx^2} + (3x-1)\frac{dy}{dx} - (4x-1)y = 0. \qquad (*)$$

Hence use the method of reduction of order to find the general solution to $(*)$.

2. Euler's method for solving

$$\frac{dy}{dx} = f(x,y), \;\; y(x_0) = y_0$$

is given by

$$y_{n+1} = y_n + hf(x_n, y_n)$$

where $x_n = x_0 + nh$ and y_n is the generated approximation to $y(x_n)$.

Show that the local truncation error for this method is proportional to h^2 as $h \to 0$.

Apply the Euler method to the equation

$$\frac{dy}{dx} = 1 - y, \;\; y(0) = 0, \qquad (+)$$

and compute the solution at $x = 0.1$ using a step $h = 0.05$.

Find the exact solution $y(x)$ to $(+)$ and, use your calculator to find $|y(0.1) - y_2|$ to four decimal places.

Comment on whether this result is consistent with the LTE found above.

3. Find the Fourier series representation over the range $-\pi < x < \pi$ of the function

$$f(x) = \begin{cases} -x^2, & -\pi < x < 0 \\ x^2, & 0 < x < \pi \end{cases}.$$

What value does the Fourier series converge to at $x = \pi$?

END OF TEST