M2S1 - ASSESSED COURSEWORK 2, 2011-12

The submission deadline is Monday 12th December, 2.00pm.
Please hand in to the Undergraduate Mathematics Student Office

1. Let Z be a standard normal random variable. Show that

$$\frac{1}{\sqrt{2\pi}}\frac{1}{t}e^{-t^2/2} > P(Z > t) > \frac{1}{\sqrt{2\pi}}\frac{t}{t^2+1}e^{-t^2/2}.$$

[*Hint.* For the right hand inequality, you might wish to consider the function

$$g(t) = P(Z > t) - \frac{1}{\sqrt{2\pi}}\frac{t}{t^2+1}e^{-t^2/2},$$

and show that this is strictly decreasing.] *[6 MARKS]*

2. Let X have the Gamma$(s,1)$ distribution. Given that $X = x$, let Y have the Poisson distribution with parameter x.

Find the moment generating function of Y and show that

$$\frac{Y - E(Y))}{\sqrt{\text{var}(Y)}} \xrightarrow{d} W, \text{ as } s \to \infty,$$

for a random variable W which you should identify. *[7 MARKS]*

3. Let X and Y be independent standard normal random variables.

Find the distributions of: (a) $X/|Y|$; (b) $X/(X+Y)$.

[*Hint.* A random variable Z has the Cauchy(μ, σ) distribution if it has probability density function of the form

$$f(z;\mu,\sigma) = \frac{1}{\pi\sigma[1 + (\frac{z-\mu}{\sigma})^2]},\ z \in \mathbb{R}.$$

Note that $Z = (Z^{-1})^{-1}$.] *[7 MARKS]*