

MATHEMATICS DEPARTMENT, IMPERIAL COLLEGE.
PROBLEM SHEET 9
GRAPHIC REPRESENTATION OF FUNCTIONS

1. Express the function

$$f(x) = \frac{x^2 + 3x + 1}{x + 3}$$

in the form

$$Ax + \frac{B}{x + 3},$$

where A and B are constants. Find the stationary points of $f(x)$ and, by evaluating the second derivative $f''(x)$ or otherwise, determine which of these are maxima or minima. Sketch the graph of $f(x)$, including clearly all the asymptotes and stationary points.

2. Let

$$f(x) = \frac{x(x+1)}{x-2}.$$

Find the stationary points of $f(x)$. By examining the sign of $f'(x)$ or otherwise, find which of these are maxima or minima. Sketch the graph of $f(x)$, indicating clearly any asymptotes.

3. Consider the function

$$f(x) = (x^2 - 1)e^{-x}.$$

- (a) Find the points where $f(x) = 0$.
- (b) Find the vertical and horizontal asymptotes of f , if any.
- (c) Use (a) and (b) to determine the sign of $f(x)$, for any x .
- (d) Find the points where $f'(x) = 0$.
- (e) Determine the local minima and maxima of f .
- (f) Sketch the graph of f .

4. Repeat the same steps (a) to (e) of Problem 3 for the function

$$f(x) = (x^2 + x - 2)e^{-2x}$$

5. Represent graphically the following function

$$f(x) = \frac{x+2}{e^{\frac{1}{x}} + 1}$$

studying its domain, intersection points, relative extremes, monotonicity and asymptotes.

6. Repeat Problem 5 for the function

$$f(x) = \frac{\ln(10x)}{x}.$$

7. Find A and B such that

$$\frac{4x-5}{x(x-1)} = \frac{A}{x} - \frac{B}{x-1}.$$

Find the stationary points of

$$f(x) = \frac{4x-5}{x(x-1)}$$

and identify them as local maxima or minima. Sketch the graph of $f(x)$ indicating the point of intersection with the x -axis and including the asymptotes.