

MATHEMATICS DEPARTMENT, IMPERIAL COLLEGE.
PROBLEM SHEET 8
LIMITS

1. Evaluate the following limits

$$(a) \lim_{x \rightarrow \pi/4} \frac{\cos(2x)}{\tan(x) - 1}, \quad (b) \lim_{x \rightarrow \pi/4} \frac{\cos(2x)}{\tan(\sqrt{x}) - 1}, \quad (c) \lim_{x \rightarrow 2} \frac{\sqrt{x+2} - 2}{\sqrt{x^3 - 4} - 2}, \quad (d) \lim_{x \rightarrow \infty} \left(\frac{x+3}{x}\right)^x.$$

2. Evaluate the following limits

$$(a) \lim_{x \rightarrow 1} \frac{(x-2)(x+2)}{(x-3)(x+1)}, \quad (b) \lim_{x \rightarrow 0} \frac{1 - \cos x}{\tan^2 x}, \quad (c) \lim_{x \rightarrow 0} x^x, \quad (d) \lim_{x \rightarrow -2} \frac{\sqrt{-2x} - 2}{x + 2}.$$

3. Given that

$$\lim_{\theta \rightarrow 0} \frac{\sin(\theta)}{\theta} = 1,$$

evaluate the following limits

$$(a) \lim_{\theta \rightarrow 0} \frac{\sin(2\theta)}{\theta}, \quad (b) \lim_{\theta \rightarrow 0} \frac{(\sin(\theta))^2}{\theta}, \quad (c) \lim_{\theta \rightarrow 0} \frac{\sin(\theta^2)}{\theta \sin(\theta)}.$$

4. State whether or not the following limits exist. If so, give the explicit value:

$$(a) \lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right), \quad (b) \lim_{x \rightarrow \infty} (\sqrt{1+x} - \sqrt{x}), \quad (c) \lim_{x \rightarrow \infty} \sin\left(\frac{1}{x}\right),$$
$$(d) \lim_{x \rightarrow 1} \frac{x-1}{x^n - 1}, \quad n \geq 2, \quad (e) \lim_{n \rightarrow \infty} \frac{2^n}{n!}, \quad n \in \mathbb{N}, \quad (f) \lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right).$$

5. Evaluate the following limits

$$(a) \lim_{x \rightarrow 0} \frac{x^2 - 1}{x^2 + x - 2}, \quad (b) \lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 + x - 2}, \quad (c) \lim_{x \rightarrow \pi/2} \sec^2(x)(1 - \sin(x)), \quad (d) \lim_{x \rightarrow \infty} x(\sqrt{x^2 + 4} - x).$$

6. Evaluate the following limits

$$(a) \lim_{x \rightarrow 0^+} \frac{\ln(x+1)}{\sin(x)}, \quad (b) \lim_{x \rightarrow 0^+} (\sin(x))^x, \quad (c) \lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}}, \quad (d) \lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{\tan^{-1}(x)}\right),$$
$$(e) \lim_{x \rightarrow 0^+} x^{\sin x}, \quad (f) \lim_{x \rightarrow \infty} \frac{x + \ln(x)}{x \ln(x)}.$$