

MATHEMATICS DEPARTMENT, IMPERIAL COLLEGE.
PROBLEM SHEET 4.b
INDEFINITE INTEGRALS

1. Evaluate the following integrals:

$$(a) \int x^n dx \text{ where } n \neq -1, \quad (b) \int (3x^2 + x - 5 - 2x^{-1} + 3x^{-3}) dx,$$
$$(c) \int (2 \sin(x) - 3 \cos(x)) dx, \quad (d) \int (e^x + e^{-x}) dx.$$

Why is $n \neq -1$ in (a)? What is the integral if $n = -1$?

2. Evaluate these integrals using trigonometric substitutions:

$$(a) \int (1 - x^2)^{1/2} dx, \quad (b) \int x^2 (1 - x^2)^{1/2} dx, \quad (c) \int (1 + x^2)^{-1} dx.$$

3. Evaluate these using integration by parts:

$$(a) \int e^x \sin(x) dx, \quad (b) \int x \ln(x) dx, \quad (c) \int x e^{ax} dx, \quad (d) \int (x - 1)^2 \ln(x) dx.$$

4. Use partial fraction to evaluate:

$$(a) \int \frac{dx}{(x-1)^2(x-3)}, \quad (b) \int \frac{2x^2+6x+10}{x^2+2x+2} dx, \quad (c) \int \frac{x+1}{x^2-x-12} dx.$$

5. Evaluate the following integrals:

$$(a) \int \frac{\sinh^{-1}(x)}{(1+x^2)^{1/2}} dx \quad (c) \int \frac{x}{\sqrt{1-x^2}} dx \quad (e) \int \frac{\sec^2(x)}{\tan(x)} dx \quad (g) \int \frac{x+1}{x^2-3x+2} dx$$
$$(b) \int \frac{dx}{1-\cos(x)} dx \quad (d) \int \frac{4x-8}{x^2-4x+5} dx \quad (f) \int x^2 \ln(x) dx \quad (h) \int \frac{1}{x \ln(x)} dx$$

6. Evaluate the following integrals:

$$(a) \int \frac{1}{1+\cos x+\sin x} dx, \quad (b) \int \sin^6(x) dx, \quad (c) \int \cos^5(x) dx.$$